

Using Numbas Explore Mode to create a Gaussian Elimination Calculator tool

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Context

Some of my colleagues in Leicester, both in maths and physics, were already using Numbas, so in Spring 2020 I started teaching myself Numbas to use it on my first year Linear Algebra course. A group of us from both subjects successfully requested the implementation of an LTI, so that we could also reliably deliver summative Numbas assessments during the pandemic. In maths, these were mostly weekly or biweekly “homework” or mid-semester class tests (contributing to the module marks), not end of year exams. I wrote around 300 questions for my Linear Algebra course during the year (mostly for formative use), with some support from a Graduate Teaching Assistant, and a little mutual testing of Numbas questions with colleagues (within maths). We also helped each other with implementation problems across maths and physics.

Motivation for the particular tool

While teaching Linear Algebra to the first year mathematics students in Leicester, I noticed that many students found it hard to understand the order of steps required in the Gauss-Jordan algorithm (also called Gaussian elimination). Students would often use suboptimal steps and go round in circles, later steps undoing their previously obtained zero entries and so on. In a conversation about this, a colleague (Jeremy Levesley, now retired) suggested that a tool which lets students just choose the necessary row operations but does the individual cell calculations for them would be helpful for more focused practice.

Implementation

When I discovered Numbas in Spring 2020, I explored the Explore Mode, and realised that the option to overwrite variables when going from one part to the next would make it possible to implement such a tool. By overwriting variables with student input or internal calculations, the linear system can be evolved through the steps of the algorithm.

Your system now looks like this:

$$\left(\begin{array}{cccc|c} 1 & 2 & \frac{2}{3} & \frac{7}{3} & \frac{1}{3} \\ -6 & -12 & 3 & 1 & 2 \\ 2 & 5 & 5 & 2 & 2 \end{array} \right)$$

Enter what you would like to calculate. You can choose one row to add to one or more of the other rows. The entries in front of “times” can be fractions. You can leave blank any entries you don’t need.

Add times row to row .

Add times row to row .

You have gone from

$$\left(\begin{array}{cccc|c} 1 & 2 & \frac{2}{3} & \frac{7}{3} & \frac{1}{3} \\ -6 & -12 & 3 & 1 & 2 \\ 2 & 5 & 5 & 2 & 2 \end{array} \right) \quad \text{to} \quad \left(\begin{array}{cccc|c} 1 & 2 & \frac{2}{3} & \frac{7}{3} & \frac{1}{3} \\ 0 & 0 & 7 & 15 & 4 \\ 0 & 1 & \frac{11}{3} & -\frac{8}{3} & \frac{4}{3} \end{array} \right)$$

What do you want to do next?

Multiply some rows

Add multiples of some row to other rows

Swap some rows

Finish

Students can enter a linear system with augmentation (with up to 5 rows, for implementation reasons). Then they enter their chosen elementary row operations and the question calculates these for them. Since the students were already interacting with other Numbas questions, the interface was familiar to them. Other Numbas features are used to warn students if the augmentation has a different size to the matrix, or to make sure that only the relevant number of input options are displayed for the current size of the system, that unrequired entries can be left empty, and some other tricks to make the tool more usable for students.

Delivery and use

In Leicester, I delivered formative Numbas questions as standalone pages (but linked via the university’s VLE), and assessments via an LTI tool through the VLE. Since the tool is not a “question” in the normal

sense, and is not marked, I offered it, and a similar one for calculating the inverse of a square matrix, as standalone webpages through the VLE, and also on [my own webpage](#). The maths students in Leicester had access to it throughout their first year in 2020/21. When I moved to TU Dresden in October 2021, I also offered it to first year physics students via personal websites. (The TU Dresden VLE does not support the required version of SCORM, so I offer all Numbas questions in Dresden via a website, just for formative use.) Of course, since there are links on my own webpage, anyone who finds it can use it there.

Feedback from students

Specifically for this case study, I collected feedback via a questionnaire in English, mostly from the physics students in Dresden, and got 25 responses (of a cohort of around 90 students). All but 1 respondent used the tools, with 24% using them once or twice, 32% using them quite a bit when the topic was new but not later, 36% using them quite a bit, also during the remainder of the course and/or for revision. One person responded that they will keep using them when they need to do such calculations.

44% of respondents said that the tools helped them to learn how to use the algorithm effectively, and 52% already knew how to use the algorithm, but the tools helped them practise other tasks from the course. 84% of respondents would recommend the use of the tools to another student studying the same topic, and 16% would maybe recommend.

I was unable to collect any feedback from the Leicester students for this case study, as I have now left Leicester. From my impressions during teaching, I believe that the cohort of Leicester maths students struggled more with the algorithm, and the tools were created with such students in mind. It would be interesting to get feedback from a similar cohort in the future, maybe through colleagues using the tool in their own courses.

Comments from the Dresden physics students include (with small corrections to their spelling):

“Made very well and easy to understand. Even though I understood the algorithms beforehand, seeing how the tool did it helped me to do my calculations more effectively from there on. On other topics seeing examples helped me understand the course contents. I very much enjoyed being able to go back to a certain step and move on differently from there while the tool saved my previous steps. “

“Nice tool to prevent calculation mistakes, easy to use.”

“It would be interesting to create such a calculator for my own. Because designing such a tool helps to gain a deeper understanding of all the linear algebra processes in general. But I think it’s quite complicated as well.”

My use of Explore Mode

The Numbas Explore Mode gave me all the features for these tools, but of course it can also be used for more standard questions. I have used it for formative questions of various types, for example to ask for the reasons for an initial multiple choice answer in a way that gives away the answer (so you would not want to have it on the same page as the first part), or to allow students to repeat a part as often as needed in the context, for example to find eigenvectors for different eigenvalues. I have found the way points are awarded too complicated to use fairly in a summative context, but Explore Mode is excellent for formative questions, and the tools discussed above.

Conclusion

The use of Numbas generally has definitely been well received by my students, especially in a formative context. I believe that these particular tools were also a success, and will be used continually over the years by new students. They are more an educational tool than a calculation tool: they help beginners to understand the algorithm, but once the relevant learning and exams have been completed, use of computer algebra systems is more efficient for solving linear systems.